QUEEN'S UNIVERSITY

DEPARTMENT OF ECONOMICS

MIDTERM EXAMINATION

Economics 390

March 2, 2012 **Instructor:** John M. Hartwick

Time Allowed: 75 minutes.

Answer 3 (three) questions. (Equal value).

1. a) The Hotelling problem "asks": given the world's S_0 tons of oil in a large tank, how does one analyze the systematic drawing down of the S_0 tons over a future interval of time? Explain the Hotelling "answer".

   b) Given demand schedule p =A - 2Q, discount rate (interest rate) r = 0.1, unit extraction cost, 2 and initial stock S_0 = 100 tons, show TWO STEPS in the solution to Hotelling's problem.

2. a) Explain "zero profit intertemporal arbitrage" in the competitive Hotelling model.

   b) Explain how the monopoly Hotelling analysis differs from the competitive industry case.

3. a) Nordhaus worked on a competitive Hotelling model, extended to incorporate different qualities of energy sources (say oil deposits). Discuss the economics of the switch from one deposit to the next.

   b) What is Nordhaus's "backstop supplier"? (Insert a backstop supply into the basic Hotelling model).

4. Intertemporal analysis of oligopoly is complicated because COMMITMENT by a "player" is an unreasonably strong assumption. Comment on "the commitment problem" in oligopoly analysis, with exhaustible resources.

TEST

Name ______________ Student Number ______________

Subject Econ390 Section ______________ Book No. 1 of 1 Books

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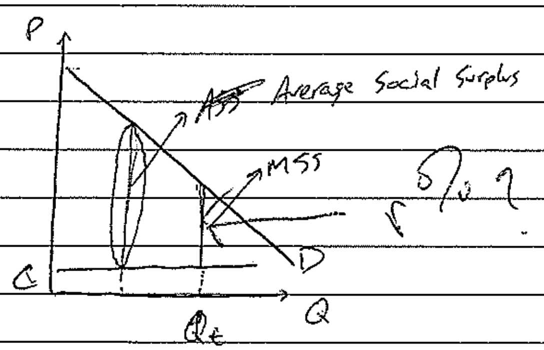
(1) a) Hotelling's solution to the optimal extraction of oil is to equalize the marginal social surplus over time periods, so that the present value of the marginal social surplus provided no opportunities for intertemporal arbitrage opportunities. This means that in every consecutive period the marginal social surplus must increase by r%. This is "Hotelling's r% rule". Starting at time T, the last time period of extraction, the marginal social surplus must be equal to the average social surplus. Since the marginal social surplus is the price less the cost for the marginal unit of production and the average social surplus is ~~the~~ greater



δ/o ?

than the marginal social surplus for all $Q > 0$ (assuming ~~time~~ constant cost, negatively-sloped demand curve), at period T to have average SS equal to marginal SS, $Q_T = 0$.

By increasing the marginal social surplus $(= P(Q) - c)$ by r% in every period working backward from time T, output in each period can be found. This production pattern is Hotelling's solution to maximizing social surplus.

"FORMULA"?

b) $P = A - 2Q \qquad r = 10\% \qquad C = 2 \qquad S_0 = 100$

Time T: $Q_T = 0 \qquad MSS = P(Q) - C = A - 2$

$T-1$: $MSS = (A-2)/(1+r) \qquad P(Q_{T-1}) - C = \frac{A-2}{1.1}$ ✓

$Q_{T-1} = \frac{A}{2} - \frac{P}{2} = \frac{A}{2} - \left(\frac{(A-2)}{1.1} + 2\right)/2 \qquad Q_{T-1} = -1 + \frac{A}{2} - \frac{A-2}{2.2}$ MINUS

$\cancel{= 2 = \frac{A}{2} - \frac{(A-2)}{1.1}}$

$T-2$: $MSS = \frac{A-2}{(1.1)^2} \qquad P(Q_{T-2}) - C = \frac{A-2}{(1.1)^2}$ ✓

$Q_{T-2} = \frac{A}{2} - \frac{1}{2}\left(\frac{A-2}{1.1^2} + 2\right) \qquad Q_{T-2} = -1 + \frac{A}{2} - \frac{A-2}{2(1.1)^2}$ MINUS

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Time Period	Output
T	0
$T-1$	$1 + \frac{A}{2} - \frac{A-2}{2(1.1)}$
$T-2$	$1 + \frac{A}{2} - \frac{A-2}{2(1.1)^2}$

Extract at t | extract at $t+1$

$P_t - c$ profit | $P_{t+1} - c$ profit.

INDIFFERENCE $\Rightarrow$

$$P_t - c = \left(\frac{1}{1+r}\right)\left(P_{t+1} - c\right)$$

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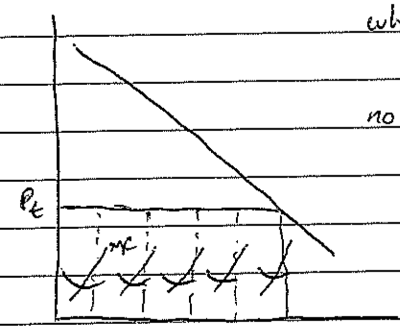
(2) a) Zero profit intertemporal arbitrage in the competitive Hotelling model is the condition that ~~what~~ ~~the production~~ results in an equilibrium production extraction pattern. When there is zero profit inter temporal arbitrage, there are no profitable opportunities available from changing the production pattern and extracting any given unit of oil in a different time period. If Hotelling's industry is represented by many small firms, each possessing only one unit of oil that must be extracted in any one given time period t $t \in (0, T)$ at cost c, there will be zero intertemporal profit abitrage if $(P_t - c) = (P_{t+1} - c)/1+r$ for all time periods. This condition will mean that the Present Value of extracting in any given time period is equal, so that firms are indifferent to which period they extract in. Competition will ensure that there is zero profit inter temporal arbitrage, which means that $(P_t - c)$ will increase at $r\%$ per year ~~to make~~ across the industry, which satisfies Hotelling's solution. In effect, firms individually maximizing profit will organize in such a way that there is no intertemporal arbitrage profit opportunities, and this outcome is necessary and sufficient to satisfy Hotelling's outcome. However, if the industry is not comprised of price-takers,

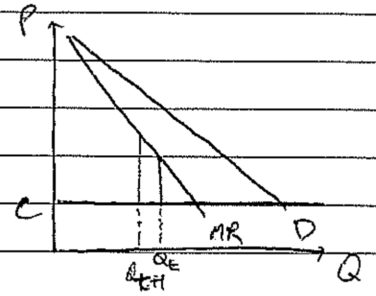
but rather L.C. Gray firms with u-shaped average cost curves,



where $Q_t = \sum^{n} q_t$ for n firms, there is no zero profit inter temporal arbitrage outcome. Firms would normally ~~will~~ increase $(P_t - MC(q_t))$ by r% per period, but since $q_T \,\cancel{\neq}\, > 0$ there ~~will~~ would be a price jump after time T if all firms increased marginal profit by r%, there is ~~will~~ a arbitrage opportunity to withhold supply until all competitors (following gray's solution) ~~and dep~~ exhaust their supplies, and then become a monopolist. Because there is an intertemporal profit arbitrage opportunity, Hotelling's necessary and sufficient condition is not met and the market outcome will not be a Hotelling solution.

PV ↑ sum

b) A monopolist seeks to ~~optimize~~ max $P(Q)Q - c(Q)$. Supposing constant unit cost of extraction c, the ~~first order condition~~ profit-maximizing condition is to increase $\left(P\left(1-\frac{1}{e}\right)-c\right)$ by $r\%$ per period. This function is marginal revenue and



for a linear demand function has a greater negative slope than the demand function. The competitive industry Hotelling solution was to increase $(P(Q)-c)$ by $r\%$ per year

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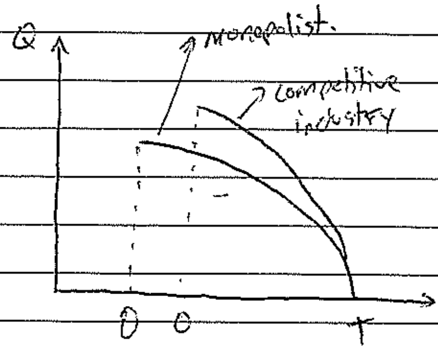
but the monopoly solution is to increase $\left(P(Q)\left[1-\frac{1}{e}\right]-c\right)$ by $r\%$ per year. ~~The~~ This means that it requires a lesser change of Q to achieve the same $r\%$ growth for a monopolist, and so

detail

a monopolist will (working backwards from T) increase production slower than a competitive industry working backwards. This means a monopolist has higher prices and a longer extraction duration than a competitive industry. Stiglitz provided a counterexample, where if $P(Q)=AQ^{-B}$



and $c=0$, the monopolist would follow the same extraction path as a competitive industry, but otherwise the monopolist would extract over a longer period than a competitive industry.

(3) a) Nordhaus' model can explain ~~heter~~ discretely heterogeneous energy sources, where the costs are the variable factor.

The model starts with the most expensive source working backwards from time T using a Hotelling model. This last firm will deplete its resources at time T but start at some time t_n (n energy sources) with some price of energy P_{t_n}. This price is the backstop price of the secondly most expensive energy source, as the final energy source will begin extracting when prices reach this level. As such, the second most expensive energy source ~~work~~ works backwards from t_n, increasing its marginal profit in each period from t_{n-1} to t_n as a Hotelling outcome. This is repeated by the next-most expensive energy source, all working backward from time T and the most expensive source.

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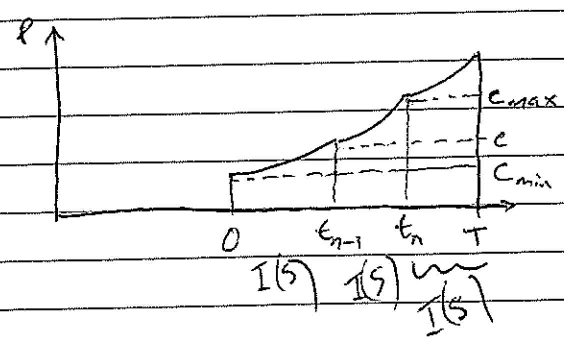
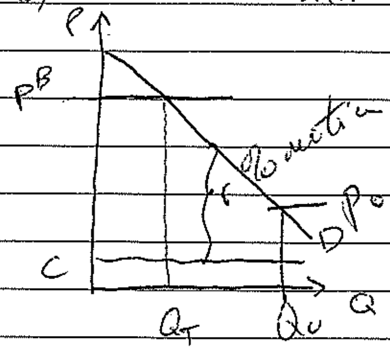
Graphically depicted below. In this manner each deposit follows a Hotelling solution, and maximizes its profit. There is zero profit intertemporal arbitrage because each deposit quality does the best it can.

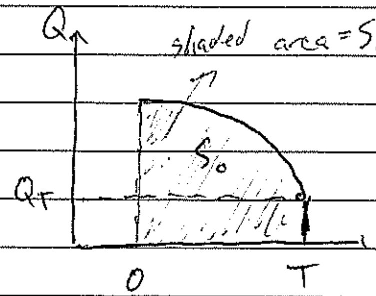


smooth price adjustment
jump in rent

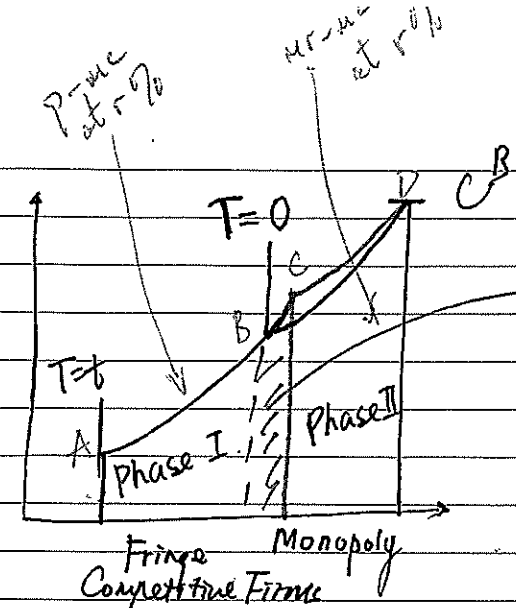
b) Nordhaus' backstop supplier is an alternative energy source that is competitive at price P_B. ~~[illegible]~~ P^B is a cap on the demand price of the energy source, so price cannot exceed P^B. If a competitive industry set $Q_T = 0$,



~~its~~ ~~price~~ ~~P~~ the market price would be P^B, rather than the demand function's corresponding price. The Hotelling solution with a backstop supplier is to set $Q_T > 0$ such that $P(Q_T) = P^B$ and work backwards from this point. If the industry failed to do this, it would be unable to increase marginal profit by r% (because at this point the price $= P^B$ and $(P^B - c)$ does not change with Q). Therefore to equalize the present value of marginal profit across time, the industry ~~the~~ will supply Q_T where P^B intersects the demand curve, and follow the extraction pattern. This means that with backstop supply ~~extraction~~ ~~with~~ depletion will occur more rapidly.



4).



Commit to AB at T=t, then fringe ~~left~~ leaves and BC is ~~no~~ less profitable than BD. Break the commitment and do not supply the shaded area, rather supply as a monopolist.

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Inorder to get rid of the fringe, the competitive firms fast, the monopolies would dump some of its resources into the market to lower the price and accelerate the extraction path of competitive firms.

mr↑ max

In phase I, the ~~mo~~ monopolist will take as a competitive firm and sell some of it's resources. It commits to ~~an~~ the extraction path AB at time t. However, ~~the~~ when time = 0 arrives, or when the competitive firms run out of ~~nest~~ their resources and are out of the energy market, following the extraction path AB to ~~BA~~ BC ~~is not as profitable~~ as a competitive firm is not as profitable as following the extraction path of BD as a monopolist. Thus, at T=0, the monopolist is tempted ~~at~~ to break the commitment, commited extraction path BC and follow the more profitable path BD in phase II.

The nature of the market makes ~~it~~ possible to exist two phases ~~of~~ extraction. The commited extraction path ~~is not~~ in the beginning of phase I is not as profitable as the ~~set~~ new extraction path in phase II.

when entering phase II. The monoplist finds it profitable to break the commitment.

It is a problem of closed loop situations where the firm has incentive to cheat and break the initial commitment.